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2.0 HISTORICAL BACKGROUND

The need for a study to estimate the marginal cost of access was created by the independent occurrence of two events in 1984:

(1) The divestiture of the Bell Operating Companies by AT&T as part of a negotiated settlement of an antitrust suit,[3] and

(2) The creation of a new service called "carrier access" to be regulated, tariffed, and provided to all carriers, as part of an on-going docket at the Federal Communications Commission (FCC) dealing with the entrance of competitors in long distance switched and private line services.[4]

Though ultimately stemming from the same causes,[5] these events exerted independent effects, in the sense that the absence of either would not have changed the outcome. Access had to be provided as a separate tariffed service, either to replace the former Bell System internal transfer pricing mechanism (to implement divestiture) or to provide equal access to all competing long distance carriers on a tariffed basis (as part of Docket 78-72).

Since carrier access service was envisioned as a regulated and tariffed service at its birth, rules for cost support for proposed rates were not far behind. Embodied in Part 69 of the Commission's Rules, the cost basis for access prices was explicitly determined to be a future test period, fully-distributed average cost. Given this environment, the need for an estimate of *marginal* costs for these services must be somewhat surprising.

There are at least three uses for these access marginal costs. First, when access prices are changed in a tariff filing, FCC rules require that the effects of those price changes on demand and on cost be taken into account in the subsequent rate calculation.[6] These parts of a tariff filing (described in Volume V of the *Access Tariff Description and Justification*) are known as "demand response" and "cost offset" estimates, respectively. The demand response calculation estimates the change in demand by services caused by the proposed "baseline" changes in access prices. The cost offset process, in turn, calculates the changes in fully-distributed service costs associated with those changes in service demands. Therefore, to apply the Part 69 methods, one must first estimate the change in *actual* total company costs; this change is then split jurisdictionally into an interstate and intrastate component following Part 67 of the Commission's Rules. The interstate component of the cost change is then further split into cost changes by service using the Part 69 Rules. The initial change in total company costs associated with a change in service demands is rightly a *marginal* cost concept, and it is this use - in interstate access tariff filings - that is the principal justification for the model.

A second justification is to provide a test for cross-subsidization in various access rate structures. Proposed rate structures for some access services involve multi-part tariffs or tapers. For situations in which different classes of users face different service prices at the margin, it is important to ensure that no class of service subsidizes another. One useful test for this is a marginal cost test, which certifies a multi-part or tapered tariff as subsidy-free (provided the lowest marginal price in the taper exceeds the marginal cost of service).

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3. United States v. AT&T Co., 552 F. Supp. 229 (D.D.C. 1982).
4. FCC Docket 78-72, *MTS and WATS Market Structure*, FCC Docket No. 78-72, 81 FCC 2d 177 (1980).
5. Namely, the emergence of competition and the technological change that made it possible.
6. Part 61.38 of the Rules requires a projection of costs for a representative 12 month period and an estimate of the effect of changed rates on traffic and revenue.

A final reason to estimate marginal costs for access services is that this industry, although currently regulated, does not possess a monopoly franchise,[7] so that service managers must be concerned with the effect of regulated access prices on the competitive position of the firm. The marginal cost of access is the limit to which competition can drive the price of access, and it is the floor above which access prices will encourage increased entry into the access market. Thus, access prices cannot substantially exceed their respective marginal costs for long periods of time, and managers of access should find current estimates of marginal cost for access services to be useful information even in a regulated environment.

2.1 Previous Studies

Econometric cost and production functions have been specified and estimated for telecommunications services for many years.[8] Interest in parameters of the cost function for former Bell System output peaked during the period of the AT&T antitrust case because a measure of the returns to scale and scope for a fully-integrated telecommunications firm was thought to be important information for social policymakers.[9] Following this work, a more relevant set of studies attempted to calculate cost offsets as part of the demand response process for intrastate rate cases.[10] Indeed, in 1983, the first annual interstate access charge filing used a marginal cost based on an earlier New York Telephone study, and subsequent annual interstate filings by the National Exchange Carrier Association (NECA) and the individual Regions or BOCs generally have relied on this result. The Commission has not criticized this methodology to date; on the contrary, the Commission explicitly used the NECA marginal cost estimate to adjust its estimate of exchange carrier costs for demand response effects in its response to the March, 1984 access tariff filings.[11]

2.2 Cost Offsets in an Interstate Access Tariff Filing

The annual interstate access tariff filing is the principal vehicle through which the Commission enforces its mandate to regulate the rate of return of the local exchange carriers on their interstate access investment. Commission rules require that the Local Exchange Carriers (LECs) forecast their demand and cost for each of the access rate elements and that a rate be set so that each LEC will earn no more than its allowed rate of return. If the baseline demand and cost forecast results in rates which are substantially different from their current levels, the Commission requires that the effects of these new rates on demand and cost be estimated, so that adjusted rates - inclusive of demand response and cost offset effects - can be calculated. These rates, then, are partially consistent in the sense that the company expects to achieve the adjusted demand volumes and adjusted

7. The Commission's *Third Report and Order* in CC Docket 78-72, (released February 28, 1983), paragraphs 110-111, in which the Commission explicitly rejects a monopoly franchise in carrier access as a market structure solution to problems created by their proposed uneconomic recovery of NTS costs.

8. L. H. Mantell, "An Econometric Study of Returns to Scale in the Bell System," Staff Research Paper, Office of Telecommunications Policy, Executive Office of the President, Washington, D.C., February, 1974, or H. D. Vinod, "Application of New Ridge Regression Methods to a Study of Bell System Scale Economies," *Journal of the American Statistical Association*, December, 1976.

9. L. R. Christensen, D. C. Christensen, and P. E. Schoech, *op. cit.*, 1983. For an opposing view, see D. S. Evans and J. J. Heckman, "Natural Monopoly," *Breaking Up Bell*, ed. D. S. Evans (New York: North Holland Publishing Company, 1983).

10. Econometric cost models for intrastate services presented in the 1985 New York State rate case: NYPSC Case No. 28961. A more accessible summary is A. Noel Doherty, "Empirical Estimates of Demand and Cost Elasticities of Local Telephone Service," *Changing Patterns in Regulation, Markets, and Technology: The Effect on Public Utility Pricing*, ed. P. C. Mann and H. M. Trebing (East Lansing: Institute of Public Utilities, 1984), pp. 115 141.

11. *Memorandum Opinion and Order*, CC Docket Nos. 83-1145 (Phase I) and 78-72 (Phase I), released May 15, 1984, paragraph 72.

costs if the baseline rates were to go into effect.[12]

Once baseline rates are established and demand responses to those rates are estimated, a change in total company costs is calculated from the marginal cost model. This change in the company budget is then separated into an interstate and intrastate component, generally using the same percentage of interstate assigned by category of cost as appears in the baseline company budget.[13] The interstate portion of these cost changes is then assigned to switched and special access categories in the same proportion as baseline costs were assigned and, within categories, cost changes are assigned to cost elements following the baseline assignment proportions.

The demand response process yields changes in demand by access element, and the calculation just described provides changes in revenue requirements by access element. This is all the information that is necessary to calculate rates for each access rate element. Rates calculated using this method have the characteristic of actual demand being equal to that estimated by the demand response program (in response to the *baseline* rates); and if these adjusted rates are in effect, the resulting revenue will be close to the revenue requirement. Note again that the demand volumes predicted by the demand response model are inconsistent with the adjusted rates (see footnote 11), but if that demand were to occur, predicted revenue would satisfy the rate of return constraint.

12. Obviously, full consistency of assumptions has not been attained; there should be further adjustment to *demand* volumes and costs to reflect the *demand adjusted* rates. In practice, the accuracy of the demand and cost elasticity estimates generally do not warrant more than one iteration of this process.

13. This is justified by the assumption (maintained throughout this procedure) that *investment* is too slow to *respond* to demand changes within the test year. Thus, investment expenses remain fixed for the test year, and separations factors that are driven by investment also remain fixed.

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3.0 SPECIFICATION, ESTIMATION, AND INTERPRETATION OF THE MODEL

3.1 Introduction

In this section we discuss the statistical methods by which we estimate the marginal cost of interstate access services and the assumptions that must hold for these methods to be valid. We present estimates of the unknown parameters of the model, statistical tests that suggest the limitations of those estimates, and an interpretation of those estimates.

3.2 Model Specification

Consider a firm (a predivestiture aggregate of Bell Operating Companies) that chooses levels of variable inputs (labor and raw materials) to produce its observed level of output at minimum cost, assuming that its capital stock is fixed at its observed level. The operative behavioral assumption is that variable cost is minimized, conditional on the level of output, prices of the variable factors of production, and the level of the capital stock that is assumed to be quasi-fixed,[14] or outside of the firm's control when levels of variable factors of production are chosen. The marginal cost estimates we will calculate will assume that the level of output, the level of the capital stock, and the prices of variable factors are exogenous.

Here, a problem is often thought to arise for a rate-of-return regulated firm. For such a firm, prices are chosen so that revenue will equal cost (including a rate of return on the allowed rate base). If our measure of output is deflated revenue, it might appear that random fluctuations in cost would be correlated with random fluctuations in measured output. In this case, one could not treat output as exogenous in the cost function and to do so would result in biased and inconsistent measures of marginal cost. However, prices are fixed using *test year* forecasts of costs and demand; prices are not updated to ensure that actual revenue identically equals actual revenue requirements. Thus, actual revenue can deviate from planned revenue because of demand perturbations and actual cost can deviate from planned cost (conditional on output) because of cost perturbations. Thus, disturbances in the cost function are not necessarily correlated with variation in deflated revenue.

We assume the cost structure of the firm can be approximated by a translog function form. This is reasonable because a translog form is a local,[15] second-order approximation to an arbitrary cost function,[16] and because there is a long history of its use in estimating telecommunications cost functions.[17] The translog variable cost function is written in the following form:

14. Over the time period of the sample, the capital stock changes; however, it is not assumed that the firm adjusts the capital stock instantaneously to minimize costs. Among the results of the study is a test of the hypothesis that the level of the capital observed is cost minimizing.

15. i.e., in the neighborhood of the observed date.

16. E. Diewert, "Applications of Duality Theory," *Frontiers of Quantitative Economics*, ed. M. Intriligator and D. Kendrick Vol. 2 (Amsterdam: North Holland Publishing Company, 1974).

17. M. Denny, C. Everson, M. Fuss, and L. Waverman, "Estimating the Effects of Diffusion of Technological Innovations in Telecommunications: The Production Structure of Bell Canada," *Canadian Journal of Economics*, February, 1981; Nadiri and Schankerman, 1981, *op. cit.*; and Christensen, Christensen and Schoech, 1983, *op. cit.*

$$\ln CV = \alpha_0 + \alpha_K \ln K + \alpha_N \ln N + \alpha_T \ln T + \alpha_Y \ln Y + \sum_i \beta_i \ln P_i \qquad \text{Equation (1)}$$

$$+ 1/2\, \delta_{KK}(\ln K)^2 + 1/2\, \delta_{NN}(\ln N)^2 + 1/2\, \delta_{TT}(\ln T)^2$$

$$+ 1/2\, \delta YY(\ln Y)^2 + 1/2 \sum_i \sum_j \gamma_{ij} \ln P_i \ln P_j + \sum_i \rho_{Ki} \ln K \ln P_i$$

$$+ \sum_i \rho_{Ni} \ln N \ln P_i + \sum_i \rho_{Ti} \ln T \ln P_i + \sum_i \rho_{Yi} \ln Y \ln P_i$$

$$+ \rho_{KN} \ln K \ln N + \rho_{KT} \ln K \ln T + \rho_{KY} \ln K \ln Y$$

$$+ \rho_{NT} \ln N \ln T + \rho_{NY} \ln N \ln Y + \rho_{TY} \ln T \ln Y.$$

where CV is variable cost; K is the level of capital input; N is the size of the network; T represents the level of technology; Y is the level of output; P_i are the prices of the variable inputs; and $\gamma_{ij} = \gamma_{ji}$.

From Shephard's Lemma,[18] the cost shares (Mi) of the two variable inputs are equal to the logarithmic derivatives of the cost function with respect to the two input prices:

$$M_i = \beta_i + \sum \gamma_{ij} \ln P_j + \rho_{Ki} \ln K + \rho_{Ni} \ln N + \rho_{Ti} \ln T + \rho_{Yi} \ln Y . \qquad \text{Equation (2)}$$

We follow standard econometric practice in appending classical disturbances to Equation (1) and the two equations embodied in Equation (2). The parameters of the cost function then can be estimated by treating the three equations in (1) and (2) as a multivariate regression and using Zellner's seemingly unrelated regressions technique.[19] Because the share Equations (2) sum identically to one, the disturbance covariance matrix of the three-equation system is singular. We treat this in estimating the system by dropping either of the share Equations (2) at the second stage of the Zellner procedure. These estimates are asymptotically equivalent to maximum likelihood methods and are invariant with respect to which factor share equation is deleted. Note, however, that hypothesis tests and estimated confidence intervals are only valid asymptotically.

This specification follows that of Caves and Christensen (1984),[20] in that a network size variable

18. Diewert, *op. cit.*

19. A. Zellner, "An Efficient Method of Estimating Seemingly Unrelated Regressions and Tests for Aggregation Bias," *Journal of the American Statistical Association*, 1962, pp. 346-368.

20. D. W. Caves, L. R. Christensen and J. A. Swanson, "Productivity in U. S. Railroads, 1951-1974," *Bell Journal of Economics*, Spring, 1980.

(N) is included as an exogenous explanatory variable. The purpose of this specification is to permit the distinction between the cost impacts of changes in output and changes in the size of the system. An increase in output that occurs without a corresponding addition to the size of the network should be less costly than an identical increase in output that is accompanied by network expansion because the existing capital stock is being used more intensely. This distinction has been embodied in two parameters that are functions of the parameters of the variable cost function (1) and (2): a returns to scale and a returns to density parameter. Returns to scale in this specification is the proportional increase in output and network size made possible by a proportional increase in all inputs, holding technology fixed; returns to density is the proportional increase in output caused by a proportional increase in all inputs, holding network size and technology fixed.

Successful estimation of Equations (1) and (2) with no further restrictions on the parameters is unlikely from our limited time series data since there are 27 unknown regression coefficients plus disturbance covariance parameters to be estimated. Moreover, there is little variation in factor shares and little independent variation in factor prices over the period of the sample. The fact that the size of the former Bell System was increasing smoothly over the period also will make it difficult to distinguish cost effects of research and development from cost changes due to increased economies of scale. The solution is to restrict the parameters of Equations (1) and (2) in plausible ways that do not contradict the data, and we will discuss such restrictions when we present the parameter estimates. *A priori*, however, we know that a cost function must be homogenous of degree one in input prices,[21] which implies the following restrictions on the parameters:

$$\sum\beta_i = 1,\ \sum\rho_{Ki} = 0,\ \sum\rho_{Ni} = 0,\ \sum\rho_{Ti} = 0,\ \sum\rho_{Yi} = 0,\ \sum\gamma_{ij} = 0, \text{ for all j.}$$

3.3 Inputs Required

To estimate the parameters of Equations (1) and (2), the following data is required for the aggregate of the predivestiture Bell Operating Companies:

- variable operating expenses
- output
- capital stock
- variable input prices:
  - labor
  - raw materials
- network size
- technology index.

Detailed development of these series from telephone company accounts is presented in Section 4.0. In this section, we present enough of an overview of the data to facilitate understanding of the results.

21. If all input prices are doubled, variable operating costs will double because only *relative* prices of inputs matter.

The basic data used in this study are similar to those used by Christensen, Cummings, and Schoech (1980) and Christensen, Christensen, and Schoech (1983)[22] with the following differences:

- a network size variable is calculated;
- observations are quarterly between 1972 to 1982, not annually; and
- the unit of observation is the aggregate of the predivestiture Bell Operating Companies, not the entire former Bell System.

The basic approach is to construct variables at the most disaggregated level possible and aggregate to the level required by Equations (1) and (2) using the appropriate index for the production function we have implicitly specified in Equations (1) and (2).[23]

Variable operating expenses were calculated as the sum of labor and raw materials costs, and as described below.

Output was based on 11 principal revenue measures including both interstate and intrastate services. The revenue categories were deflated by appropriate price indexes to obtain 11 quantity indexes which were then combined into a Tornqvist index of aggregate former Bell System output.

The *capital stock* series was calculated by applying the perpetual inventory method to twenty different types of owned tangible assets. Two surveys of former Bell System capital stocks (in 1972 and 1979) provided benchmark measures of the capital stock and replacement rate in each account. For each of the twenty categories, investment and depreciation series were constructed; when combined with the benchmark data, this provided a capital stock in service time series for each category. These stocks, asset prices, and replacement rates, combined with former Bell System cost of capital and tax data, were used to calculate capital service price weights.[24] These weights then were used to calculate a Tornqvist index of aggregate former Bell System capital stock.

Real labor input was calculated by constructing a Tornqvist index using wage rates and aggregate hours worked for each of several major employee groups. The *labor price index* was calculated by dividing total labor compensation by this estimate of the quantity of labor services employed. Labor costs were calculated as the product of the labor price index and the measure of real labor input.

Total raw materials costs were calculated as a residual by subtracting labor costs, operating rents, and depreciation and amortization expenses from total operating expenses. The *raw materials price index* was assumed to be proportional to the GNP price deflator.

Network size was measured as the number of main and equivalent telephones in service.

22. L. R. Christensen, D. Cummings, and P. E. Schoech, "Productivity in the Bell System, 1947-1977," paper presented at the Eight Annual Telecommunications Policy Research Conference, April 1980; and Christensen, Christensen, and Schoech (1983), *op. cit.*

23. Subsection 4.2

24. Section 4.0

The *technology index* was taken to be the Poisson-weighted 15-year distributed lag in real research and development expenses in the former Bell System which was developed by Vinod (1976).[25]

Quarterly observations for all of these series were calculated from 1972 to 1982; the data are presented in Figure 4-4 in Subsection 4.5, Data Sources.

3.4 Outputs from the Model

Fitting the cost and factor share Equations (1) and (2) to the time series data discussed in Subsection 3.2 provides estimates of the unknown parameters of the model (the Greek letters in Equations (1) and (2)). In addition, test statistics can be calculated from these estimates, which suggest whether or not the assumptions we have made in the model specification and estimation process are contradicted by the data.

Recall that the purpose of estimating this model is to obtain estimates of the marginal cost of various access services. The coefficient of output in Equation (1) is an estimate of the elasticity of operating cost with respect to output; multiplying it by the ratio of measured output to operating cost yields an aggregate marginal cost (i.e., the dollar change in cost associated with a dollar change in output). Subsection 5.1 discusses additional assumptions and a method under which that change in cost can be attributed to changes in outputs of the different component services of total output. Note that aggregate marginal cost - as defined above - will vary with the ratio of output to cost and thus vary over time. On the assumption that recent experience is most relevant, we will calculate marginal costs as they were in 1982. Obviously, aggregate marginal costs could be inputted to current periods using current output-cost ratios, but since the approximation properties of the translog cost function are only valid locally, we will not deviate from the range of data observed.

3.5 Estimation of the Model

As pointed out earlier, estimation of the cost function in its most general form (Equation (1)) is unlikely to succeed because there are too many different parameters to estimate independently from limited time series data. Accordingly, the strategy in estimating the model is to find the most general restricted specification of the model that has the following two characteristics. First, the implied shape of the estimated cost function accords with economic theory throughout the sample period. Operationally, we shall take this to mean that throughout the sample period:

(1) costs should increase with network, output, and factor prices;

(2) costs should decrease with improvements in technology;

(3) the shape of the cost function should not vary wildly within the sample period; and

(4) the elasticity of variable costs with respect to the capital stock should approximate the negative of the ratio of capital costs to variable costs.[26]

Second, the parameter restrictions necessary to yield the first characteristic are *a priori* plausible and are not contradicted in statistical tests based on the observed data.

25. H. D. Vinod, "Application of New Ridge Regression Methods to a Study of Bell System Scale Economies," *Journal of the American Statistical Association*, December, 1976.

26. This is necessary for the quasi-fixed capital stock to be at its optimal value. In the sample, the ratio of capital costs to variable operating expenses is roughly constant at 0.9.

Point estimates and standard errors for Equation (1) - constrained only for first degree homogenelty - are given in Table 3-1 (Tables 3-1 to 3-8 are presented at the end of this section). Point estimates are imprecise and the estimated cost function is very unstable in the sense that there are some quarters in which aggregate marginal costs are negative or in which operating costs rise as technology improves. Quarterly estimates of returns to scale range from -9.17 to +44.74.

The most restrictive version of Equation (1) that remains economically plausible is essentially a Cobb-Douglas cost function, which includes only the first-order terms of each variable. This allows returns to scale to differ from one but does not permit quarterly variation in the degree of scale economies. Further, it restricts the elasticity of substitution[27] to unity.

Parameter estimates for this specification are presented in column [2] of Table 3-2. Eight increasingly general specifications are presented here, in which the model (1) is limited to first-order terms, second-order terms of single variables, and second-order cross-price terms. Virtually all nine of the cost functions estimated in Table 3-2 meet the reasonability tests just described. Specification [2] has correct signs, four parameter estimates that are significantly different from zero, and capital stock elasticity with approximately the right magnitude.

The results for specification [3] suggest that a second-order capital term significantly improves the explanatory power of the model. Unfortunately, inclusion of the second-order capital term causes the cost elasticity of the capital stock to vary implausibly, from -0.99 in 1972 to -0.53 in 1982. The second-order terms introduced in specification [4] are statistically insignificant. Specification [5] suggests that the second-order technology term may be statistically important. Moreover, the results imply that the change in costs with respect to technology rises from -0.6 in 1972 to -0.3 in 1982 which, in turn, implies that returns to research and development, though still positive in 1982, have fallen over the sample period. Second-order price terms all appear to be important in specification [6].

Specifications [7] through [10] explore combinations of second-order terms. Specification [7] includes first-order terms and second-order capital stock and price terms. However, as observed in specification [3], inclusion of a second-order capital term yields a cost function that is implausibly unstable over time with respect to the capital stock. Specification [8] includes second-order network, output, and price terms, but, as in specification [4], the network and output terms are statistically insignificant. Specification [9] combines second-order technology and price terms and has roughly the same result and interpretation as specification [5]. The second-order capital term is included along with technology and price terms in specification [10] and exerts its previously-observed destabilizing influence.

In summary, it appears that the inclusion of a second-order capital term in any specification leads to poor performance of the estimated cost function and that second-order network and output terms do not increase the explanatory power of any specification of the model. On the positive side, second-order price and technology terms provide significant increases in explanatory power and plausible results. On this basis, specification [9] appears to be superior to its alternatives in Table 3-2.

Table 3-2 provides estimates of returns to scale and returns to density, calculated at the mean of the sample. These can be calculated directly from the parameters of the variable cost function by the formulas:

27. A measure of the responsiveness of factor shares to relative changes in factor prices.

$$SCE = [1 - (\partial \ln CV/\partial \ln K)]/(\partial \ln CV/\partial \ln Y + \partial \ln CV/\partial \ln N)$$

$$DNS = [1 - (\partial \ln CV/\partial \ln K)]/(\partial \ln CV/\partial \ln Y),$$

where SCE and DNS are returns to scale and returns to density, respectively. Scale economies range from 0.77 to 1.03, but throughout Table 3-2, they never differ from one by an amount that is statistically significant. Hence, we estimate a further restriction of specification [9], which imposes constant returns to scale through the constraint:

$$\alpha_N = 1 - \alpha_K - \alpha_Y.$$

Imposing this constraint causes insignificant shifts in the point estimates of the parameters but yields generally smaller estimated standard errors: compare columns [9] and [11] in Table 3-3.

Model [11] in Table 3-3 is our preferred model; the marginal costs we will use for further work will be based on its parameter estimates. It has the following desirable economic properties:

- costs increase with output, network size, and input prices
- it is homogeneous of degree one in prices
- costs decrease with improving technology
- the observed capital stock is relatively close to the cost minimizing level implied by model [11]
- the cost function is concave with respect to input prices.

Statistically, all parameters in the model are significantly different from zero. The fit of the system of equations is more than adequate, as shown in Table 3-4. The Durbin-Watson statistics reject the presence of first-order serial correlation of the residuals. The similarity in statistical properties of the factor share equations is due to the singularity of the disturbance covariance matrix which, in turn, is due to the fact that the shares must sum identically to one. Fitted values and residuals for the three equations are presented in Table 3-5. Finally, to examine the plausibility of the totality of the parameter restrictions embodied in specification [11], we compare it against the most general model (specification [1], which embodies only homogeneity). The appropriate statistical technique is an F test of the hypothesis that the additional restrictions in specification [11] are valid; it compares the sum of squared residuals in specification [11] with those in specification [1] to determine the loss in fit produced by imposing restrictions that may or may not be consistent with the data. The F statistic equals 0.98, which is substantially less than a conservative critical value of 2.20, so that we are unable to reject the restrictions on specification [1] that yield specification [11].

3.6 Interpretation of the Results

Table 3-2 gives marginal costs of switched and special access for 1982, which are implied by the various models. This calculation of the marginal costs for various components of total output is based on the formula:

$$MC_i = \frac{\partial \ln C_v}{\partial \ln Y} \times \frac{\partial \ln Y}{\partial \ln Y_i} \frac{C_v}{Y_i},$$

where $(\partial \ln C_v/\partial \ln Y)$ represents the elasticity of variable cost with respect to aggregate output;

$(\partial \ln Y/\partial \ln Y_i)$ represents the elasticity of aggregate output with respect to output i; C_v represents total variable cost in 1982, and Y_i represents the level of output i in 1982 (represented in conversation minutes for switched access and dollar revenues for private line).

The elasticity of cost with respect to output comes directly from the econometric model estimated and is 0.38 in specification [11]. The elasticity of aggregate output with respect to output i comes directly from the aggregator function used to construct aggregate output. It is equal to:

$$q_{it}Y_{it}/\sum_j q_{jt}Y_{jt},$$

where q_{jt} is the set of prices used to construct Y. In the case where indexes of prices received are used in aggregation, this elasticity is the revenue share of class i.

Application of these formulas to calculate 1982 marginal costs for switched and special access services is outlined in Tables 3-6 and 3-7. Note that the marginal cost per dollar of revenue in 1982 is the same for switched and special access services. This is a consequence of the assumption used in constructing our measure of aggregate output that marginal costs are proportional to prices across services. Thus an *assumption* of our model is that the additional cost associated with an additional dollar of revenue is the same for all services.

This formula - and the interpretation of the calculations as marginal costs of interstate access services - depend on two critical assumptions. First, we must assume that the interstate revenue of the aggregate of the predivestiture Bell Operating Companies during this period, represents revenue derived from services that are similar to interstate access services provided today. This may be approximately valid, since the system of access charges set up in FCC Docket 78-72 was constructed to mirror the predivestiture division of revenues process as closely as possible. Second, the construction of the aggregate output index implicitly assumes that the marginal cost of each of the 11 categories of output is proportional to its price. Since services' prices were set by fully-distributed costs, it is unlikely that this condition would hold exactly. Indeed, specific parts of the price of interstate toll were derived from an assigned recovery of some of the costs of local service, so that it may be that the ratio of local-to-toll marginal costs was higher than the ratio of local-to-toll prices. To determine the sensitivity of our results to this possibility, we allow the ratio of local-to-toll marginal costs to exceed the ratio of local-to-toll rates by 10, 20, 30, 40 and 50 percent, recalculate aggregate output, re-estimate specification [11] and recalculate the associated marginal costs.

Sensitivity results are presented in Table 3-8. Our preferred model [11] implies that, in 1982, the predivestiture Bell Operating Companies incurred 5.15 cents in operating costs for an additional interstate switched access conversation minute and 20.68 cents for every additional dollar of revenue: interstate private line revenue or interstate switched access revenue, in particular. In an extreme violation of the assumptions underlying our output aggregation, the switched access marginal cost would fall to 4.39 cents per minute and the special (or switched) access marginal cost per dollar of revenue would fall to 17.61 cents. A 50 percent violation of our assumptions gives rise to roughly a 15 percent change in our marginal cost estimates.

Recall from Section 2.0 the three uses to which results from this model are to be put:

(1) to calculate changes in costs caused by rate changes for demand response in interstate access tariff filings,

(2) to set a floor for access service prices to ensure that all subcategories of the regulated service recover at least their marginal cost, and

(3) to estimate the level to which competition can drive prices for interstate access.

For the first activity, marginal cost per dollar of service revenue is the appropriate measurement. In Section 5.0, we take the point estimate for the model (for 1982), adjust it for a future test year, and show how it is applied in a tariff filing. For the second and third activities, the relevant marginal cost (for switched access) is the cost per minute calculation, and that requires a few adjustments, descriptions of which follow.

First, in calculating the marginal cost per minute in Table 3-6, we used an average cost per minute figure derived from switched *conversation* minutes. Roughly speaking, a conversation minute is equivalent to two access minutes: one at the originating and one at the terminating location of the call.[28] Hence, the marginal cost of a switched access minute (in 1982) implied by the model is approximately 2.58 cents per access minute with a standard deviation of 0.76 cents. Therefore, the 95 percent confidence interval for the marginal cost of a switched *access* minute implied by the model is (1.09,4.06) cents per access minute.

Some further adjustments to this estimate are necessary to compensate for differences between the predivestiture BOC interstate service offering in 1982 and BOC interstate access service today. The 2.58 cents per access minute marginal cost includes the following services, which are not included in switched access today:

(1) billing and collection,

(2) operator services, and

(3) directory assistance.

The fully-distributed costs of these services generally exceeds one cent per access minute, which - if it approximates 1982 marginal costs - would lower our estimate of the marginal cost of a current switched access minute (in 1982) to roughly 1.5 cents per minute. Further, even less quantifiable adjustments must be made to compensate for changes in technology since 1982 (predominantly digital switching and fiber transmission) and for costs associated with predivestiture BOC network facilities that were transferred to AT&T at divestiture. The effect of both of these adjustments is to further reduce the 1.5 cent marginal cost estimate.

28. It differs from two primarily because of call setup time, busy calls, and other incomplete calls whose access time is charged to the interexchange carrier at the originating end.

Table 3-1. Estimated Parameters of the Translog Variable Cost Model

α_o	-22953 (29835)	ρ_{NL}	.290 (1.06)
α_K	241 (951)	ρ_{NM}	-.290 (1.06)
α_N	2717 (5436)	ρ_{TL}	-.028 (.447)
α_T	-1857 (1776)	ρ_{TM}	.028 (.447)
α_Y	6.47 (1168)	ρ_{YL}	-.268 (.261)
β_L	-3.14 (11.0)	ρ_{YM}	.268 (.261)
β_M	4.14 (11.0)	ρ_{KN}	-.130 (94.6)
δ_{KK}	52.3 (48.9)	ρ_{KT}	-28.4 (31.0)
δ_{NN}	66.3 (556)	ρ_{KY}	48.3 (23.4)
δ_{TT}	-64.0 (58.7)	ρ_{NT}	170 (148)
δ_{YY}	19.2 (30.7)	ρ_{NY}	-80.0 (125)
γ_{LL}	-.026 (.142)	ρ_{TY}	-13.2 (29.9)
γ_{LM}	.026 (.142)		
γ_{MM}	-.026 (.142)		
ρ_{KL}	.223 (.248)		
ρ_{KM}	-.223 (.248)		

Table 3-2. Parameter Estimates for Nine Variations of the Translog Cost Function

(standard errors in parentheses)

Parameter	[2] 1st Order Terms	[3] 2nd Order Capital	[4] 2nd Order Output, Network	[5] 2nd Order Technology	[6] 2nd Order Prices
α_K	-0.94 (0.22)	-22.04 (4.85)	0.81 (0.19)	-0.83 (0.19)	-0.94 (0.22)
α_N	1.60 (0.89)	2.02 (0.75)	3.45 (61.24)	2.07 (0.76)	1.55 (0.91)
α_T	-0.21 (0.25)	-0.50 (0.22)	0.40 (0.22)	-3.39 (0.79)	-0.18 (0.26)
α_Y	0.33 (0.23)	0.24 (0.19)	6.81 (16.82)	0.24 (0.19)	0.34 (0.23)
β_L	0.77 (0.00)	0.77 (0.00)	0.77 (0.00)	0.77 (0.00)	0.74 (0.00)
β_M	0.23 (0.00)	0.23 (0.00)	0.23 (0.00)	0.23 (0.00)	0.26 (0.00)
δ_{KK}		0.97 (0.22)			
δ_{NN}			0.30 (3.38)		
δ_{TT}				0.63 (0.15)	
δ_{YY}			0.31 (0.73)		
γ_{LL}					0.08 (0.03)
γ_{LM}					0.08 (0.03)
γ_{MM}					0.08 (0.03)
Scale	1.00 (0.31)	0.77 (0.17)	0.81 (0.18)	0.79 (0.17)	1.03 (0.33)
Density	5.80 (4.28)	7.12 (5.91)	8.81 (9.20)	7.58 (6.40)	5.72 (4.25)
1982 Marginal Costs (cents)					
Switched Access	4.53	3.30	4.17	3.26	4.59
Private Line	18.18	13.25	16.74	13.11	18.44

Table 3-2. Parameter Estimates for Nine Variations of the Translog Cost Function (continued)

(standard errors in parentheses)

Parameter	[7] 2nd Order Capital, Prices	[8] 2nd-Order Network, Output Prices	[9] 2nd-Order Technology Prices	[10] 2nd-Order Capital, Technology Prices
α_K	-22.81	-0.81	-0.83	-28.02
	(4.92)	(0.19)	(0.19)	(35.47)
α_N	2.00	-0.81	2.05	2.00
	(0.75)	(0.19)	(0.76)	(0.76)
α_T	-0.48	-0.39	-3.54	0.23
	(0.22)	(0.23)	(0.79)	(4.98)
α_Y	0.24	-7.72	0.24	0.24
	(0.19)	(16.93)	(0.19)	(0.19)
β_L	0.73	0.73	0.73	0.74
	(0.00)	(0.01)	(0.00)	(0.01)
β_M	0.27	0.27	0.27	0.26
	(0.00)	(0.01)	(0.00)	(0.01)
δ_{KK}	1.00			1.24
	(0.23)			(1.62)
δ_{NN}		0.21		
		(3.41)		
δ_{TT}			0.66	-0.16
			(0.15)	(1.08)
δ_{YY}		0.35		
		(0.74)		
γ_{LL}	-0.09	-0.09	0.09	-0.09
	(0.03)	(0.03)	(0.03)	(0.03)
γ_{LM}	0.09	0.09	0.09	0.09
	(0.03)	(0.03)	(0.03)	(0.03)
γ_{MM}	-0.09	-0.09	-0.09	-0.09
	(0.03)	(0.03)	(0.03)	(0.03)
Scale	0.77	0.81	0.80	0.77
	(-0.17)	(0.18)	(0.18)	(0.18)
Density	7.28	8.94	7.59	7.09
	(6.27)	(9.52)	(6.46)	(6.04)
1982 Marginal Costs				
Switched Access	3.21	4.31	3.25	3.27
Private Line	12.90	17.30	13.06	13.15

Table 3-3. Imposition of Constant Returns to Scale Upon the Second Order Technology and Prices Model

(standard errors in parentheses)

Parameter	[9] Second-Order Technology Prices	[11] Second-Order Technology Prices SCE = 1.00
α_K	-0.83	-0.79
	(0.19)	(0.18)
α_N	2.05	1.40
	(0.76)	(0.26)
α_T	-3.54	-3.21
	(0.79)	(0.71)
α_Y	0.24	0.38
	(0.19)	(0.11)
β_L	0.73	0.73
	(0.00)	(0.01)
β_M	0.27	0.27
	(0.00)	(0.01)
δ_{TT}	0.66	0.63
	(0.15)	(0.15)
γ_{LL}	-0.09	-0.09
	(0.03)	(0.03)
γ_{LM}	0.09	0.09
	(0.03)	(0.03)
γ_{MM}	-0.09	-0.09
	(0.03)	(0.03)
Scale	0.80	1.00
	(0.18)	(0.00)
Density	7.59	4.69
	(6.46)	(1.67)
1982 Marginal Costs		
Switched Access	3.25	5.15
Private Line	13.06	20.68

Table 3-4. Statistical Fit of Preferred Model

Equation	Multiple R^2	Sum of sq. resid.	Durbin-Watson
Cost Function	0.99900	.006755	1.89524
Labor Share	0.20418	.012228	1.778
Materials Share	0.20431	.012228	1.778

Table 3-5. Fitted Values and Residuals of Preferred Model

Quarter	Log Fitted Cost	RESIDL	Fitted LSFT Labor Share	RESIDL	Fitted MSFT Materials slme	RESIDL
19721	21.54712	.006348	.781387	-.011759	.218613	.011755
19722	21.56862	.009277	.779617	-.009314	.220383	.009320
19723	21.60170	.003632	.778083	.018404	.221917	-.018410
19724	21.63797	-.003128	.776951	.003254	.223049	-.003256
19731	21.66249	-.018539	.776032	-.006693	.223968	.006691
19732	21.69278	-.021469	.775403	-.010984	.224597	.010983
19733	21.71394	.000961	.774902	.-.012632	.225098	.012633
19734	21.74301	.025253	.774281	.007498	.225719	-.007497
19741	21.76425	-.009094	.774322	-.003454	.225678	.003454
19742	21.79451	-.006393	.774206	-.012944	.225794	.012941
19743	21.82275	.004410	.774209	.011893	.225791	-.011890
19744	21.85335	.013590	.773884	.029888	.226116	-.029885
19751	21.87430	.002396	.772720	.007039	.227280	-.007036
19752	21.90839	-.001282	.771259	-.005752	.228741	.005746
19753	21.93861	.011246	.770289	.027595	.229711	-.027597
19754	21.97308	.002563	.769169	.025977	.230831	-.025975
19761	22.00569	-.004944	.767429	.006943	.232571	-.006946
19762	22.03801	-.016602	.765890	-.007693	.234110	.007697
19763	22.06712	-.007416	.765123	.027854	.234877	-.027849
19764	22.08803	.009054	.765345	.009797	.234655	-.009793
19771	22.11345	-.006088	.764275	-.009834	.235725	.009833
19772	22.14040	-.008179	.764447	-.014927	.235553	.014931
19773	22.16646	.002029	.764287	-.005453	.235713	.005454
19774	22.20280	.006485	.764045	.031881	.235955	-.031882
19781	22.23035	.003845	.763611	.006662	.236389	-.006668
19782	22.26491	.000214	.764319	-.030810	.235681	.030806
19783	22.29639	-.004181	.764161	.017793	.235839	-.017789
19784	22.33058	.017868	.763685	-.012985	.236315	.012986
19791	22.37057	-.002991	.763155	.004330	.236845	-.004334
19792	22.40440	.002472	.762665	-.029321	.237335	.029322
19793	22.44028	-.003845	.762244	.007930	.237756	-.007937
19794	22.46710	.214328	.761698	.005423	.238302	-.005421
19801	22.50232	-.006348	.761669	-.012157	.238331	.012157
19802	22.53494	-.014359	.761958	-.028206	.238042	.028208
19803	22.56697	-.008194	.761558	-.010136	.238442	.010140
19804	22.59413	-.005630	.761403	.012347	.238597	-.012340
19811	22.62128	-.232608	.761252	-.016752	.238748	.016745
19812	22.65620	.031662	.760260	-.040153	.239740	.040148
19813	22.69435	.015305	.758738	-.006127	.241262	.006122
19814	22.73502	.009735	.755770	.004668	.244230	-.004663
19821	22.77666	.005890	.752274	.017889	.247726	-.017896
19822	22.81931	.012131	.749198	-.002513	.250802	.002516
19823	22.85818	-.023743	.745868	.014944	.254132	-.014948
19824	22.88869	-.005920	.742702	.006770	.257298	-.006764

Table 3-6. Calculation of Marginal Cost for Switched Access

1.	Elasticity of Operating Expense with Respect to Total Output	.3810
2.	Elasticity of Total Output with Respect to Interstate Switched Access (MTS and WATS)	.2361
3.	Elasticity of Operating Expense with Respect to Interstate Switched Access	.0900
4.	Total Variable Costs - 1982 (millions)	$32,998
5.	Number of Switched Access Conversation Minutes - 1982 (millions)	57,688
6.	Marginal Operating Cost of a Switched Access Minute - 1982	$.0515
7.	Standard Deviation of Marginal Cost of Switched Access Minute	$.0152
8.	Switched Services Revenue - 1982 (millions)	$60,769
9.	Marginal Operating Cost per Dollar of Switched Revenue	$.2068
10.	Standard Deviation of Marginal Cost per Dollar of Switched Revenue	$.0610

Table 3-7. Calculation of Marginal Cost of Private Line

1.	Elasticity of Operating Expense with Respect to Total Output	.3810
2.	Elasticity of Total Output with Respect to Interstate Private Line Output	.0395
3.	Elasticity of Operating Expense with Respect to Interstate Private Line Output	.0150
4.	Total Variable Costs - 1982 (millions)	$32,998
5.	Total Revenues - 1982 (millions)	$2,401
6.	Marginal Operating Cost per Dollar of Revenue - 1982	$.2068
7.	Standard Deviation of Marginal Cost	$.0610

Table 3-8. The Scale-Constrained Second Order Price and Technology Model: Variations in Output Prices from Marginal Costs

(standard errors in parentheses)

Parameter	[11] No Relative Variation	[11a] 10% Relative Variation	[11b] 20% Relative Variation	[11c] 30% Relative Variation	[11d] 40% Relative Variation	[11e] 50% Relative Variation
α_K	-0.79	-0.78	-0.78	-0.78	-0.77	-0.77
	(0.18)	(0.18)	(0.19)	(0.19)	(0.19)	(0.19)
α_N	1.40	1.39	1.38	1.37	1.37	1.36
	(0.26)	(0.26)	(0.27)	(0.27)	(0.27)	(0.28)
α_T	-3.21	-3.22	-3.23	-3.24	-3.25	-3.26
	(0.71)	(0.71)	(0.72)	(0.72)	(0.72)	(0.72)
α_Y	0.38	0.39	0.40	0.40	0.41	0.41
	(0.11)	(0.12)	(0.12)	(0.12)	(0.13)	(0.31)
β_L	0.73	0.73	0.73	0.73	0.73	0.73
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
β_M	0.27	0.27	0.27	0.27	0.27	0.27
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
δ_{TT}	0.63	0.63	0.63	0.63	0.64	0.64
	(0.15)	(0.15)	(0.15)	(0.15)	(0.15)	(0.15)
γ_{LL}	-0.09	-0.09	-0.09	0.09	-0.09	0.09
	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)
γ_{LM}	0.09	0.09	0.09	0.09	0.09	0.09
	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)
γ_{MM}	-0.09	-0.09	-0.09	-0.09	-0.09	-0.09
	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)
Scale	1.00	1.00	1.00	1.00	1.00	1.00
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Density	4.69	4.57	4.49	4.42	4.37	4.32
	(1.67)	(1.65)	(1.64)	(1.63)	(1.63)	(1.64)
1982 Marginal Costs						
Switched Access	5.15	5.00	4.83	4.68	4.53	4.39
Private Line	20.68	20.07	19.42	18.80	18.19	17.61

4.0 DATA DEVELOPMENT INSTRUCTIONS: HOW TO ESTIMATE TELEPHONE OPERATING COMPANIES' MARGINAL COSTS OF LONG-DISTANCE SERVICE

4.1 Introduction

There are two major requirements for statistical analysis of telephone company costs. First, key economic parameters must be developed from the initial database. Second, a sophisticated knowledge of econometric techniques must be applied in the analysis of the data's implications.

This section addresses the first issue, that of data development. It describes the actual procedures used to prepare key variables for the 1985-1986 study of the BOCs' aggregated costs.

4.2 The Issue of Aggregation

Costs depend on the prices of the capital, labor, and materials inputs used to provide telephone services, as well as upon the levels of characteristics of these services. Because there are many subcategories of each input, and because it would be unsuitable to include these numerous subcategories in a regression equation directly, it is necessary to develop aggregate measures of the prices and quantities of the three input categories and of output.

The aggregation is accomplished through a Tornqvist indexing procedure.[29] Through this procedure it is possible, for example, to aggregate different types of labor into a single labor input, and to aggregate different telephone services into a single output. For a particular input or output, this index is developed through use of the following equations:

$$\ln(X_t/X_{t-1}) = \sum W_{i,t} \ \ln(X_{i,t}/X_{i,t-1})$$

$$W_{i,t} = (W_{i,t} + W_{i,t-1})/2$$

$$W_{i,t} = P_{i,t}X_{i,t}/(\sum P_{jt}X_{jt}).$$

where $X_{i,t}$ is the quantity of input (or output) i; $P_{i,t}$ is the price of i, and X_t is the aggregate quantity index.

Once an arbitrary value is assigned to the index for some base period, the values of the index in all other periods may be determined from the equations.

An important feature of the Tornqvist index is that it is exactly consistent with the homogeneous translog cost function used by Christensen Associates in the econometric analysis of telephone company costs. Like the translog cost function, this index does not require inputs to be perfect substitutes: The index allows for the possibility that companies will decrease use of particular input as the relative price of that input increases.

4.3 Basic Data Required

Ideally, data would be collected in forms immediately suitable for aggregation into the variables required for regression analysis. In general, however, data are collected for other purposes, and must be manipulated extensively to create the precursors of the regression variables.

29. Tornqvist (1936). Diewert (1976) gives a good summary of the properties of the Tornqvist index.

Consequently, the following data series were collected and used to generate the aggregate variables required for the econometric model:

For labor variables:

- total wage earnings
- wage earnings allocated to plant construction
- wage earnings allocated to plant removal.

For materials variables:

- total operating expenses
- operating rents
- total depreciation and amortization expense
- GNP price deflator.

For the output variable:

- price indexes, by type of service
- revenues, by type of service.

For the network variable:

- number of main equivalent telephones in the system.

4.4 Development of Variables

4.4.1 Labor Variables

Statistical analysis of the cost function requires development of two key labor variables: one that measures real labor input, and another that measures the price of labor. These were developed in five steps.

In the first step, the quarterly payroll expense for the BOCs and for Long Lines (LL) was calculated as that portion of total wage earnings that is purely an operating expense. This was accomplished by subtracting those costs that are related to capital from total wage earnings:

Quarterly Payroll for BOCs (or LL) = total wage earnings − wage earnings allocated to plant construction − wage earnings allocated to plant removal.

The foregoing measure of payroll corresponds to that appearing in the *Bell System Statistical Manual* (1982) (hereafter referred to as *BSSM*), page 703, column 1. Wage earnings used in this development were obtained from Monthly Report No. 23 (MR23).

Total compensation is developed by adding the fringe benefits to the basic payroll expense. To estimate total compensation for Long Lines and BOCs, it was assumed that the ratio of payroll to total compensation for the two entities is the same as for the entire former Bell System. This ratio can be derived from data appearing in *BSSM* on page 704, column 1.

Furthermore, total compensation must be distributed over the quarters in each year. This is done by assuming the ratio of payroll to total compensation holds constant for each quarter in the year.

With total compensation determined, this figure is split between a price index and a quantity index. First, annual prices and quantities for the former Bell System are constructed. These parameters then are used to develop quarterly indexes for the BOCs and Long Lines. The former

Bell System quantity index of labor is the product of total hours worked and a composition index, the latter reflecting changes in the occupational and experience mix of employees. For the years 1972 to 1979 both are taken from Christensen, Christensen and Schoech (1981). The composition index for 1980-1982 was constructed from data on the former Bell System employment found in the Bureau of Labor Statistics (BLS) Wage Surveys. Hours worked for 1980-1982 was obtained from a regression equation linking former Bell System average hours per employee to average hours worked in the telecommunications industry, as reported by BLS, and from the count of former Bell System employees. With annual hours worked and composition determined, an annual quantity index and an annual price index can be determined from former Bell System annual total compensation.

The annual price index of labor is converted to a quarterly price index through interpolation. This quarterly price index of labor was applied to both the BOCs and Long Lines, so that quarterly total compensation could be separated into a price and quantity of labor.

Subsection 4.4.2 contains a detailed description of the development of the labor quantity and price indexes. The variable names referred to in this description are those used in the development of the cost model described in this study. This information is provided for users who require a detailed knowledge of the actual steps taken to calculate the labor variables and can be omitted with no loss in continuity.

4.4.2 Detailed Procedures Used to Develop Labor Quantity and Labor Price Indexes

Use the following steps to develop labor quantity and labor price indexes:

1. Update annual total labor compensation, hours worked, and composition index for the former Bell System* [BS], the Bell Operating Companies [BOCs], and Long Lines [LL]. These data reflect 1972 through 1982. Also develop annual total labor input [labor quantity].

   Total Labor Compensation [BS] = A4012Y** from *BSSM*
   Hours Worked [BS] = A4101Y from Christensen, Christensen, Schoech
   Composition Index [BS] = A4102Y from Christensen, Christensen, Schoech
   Total Labor Input [BS] = Hours Worked × Composition Index
   Total Labor Input [BS] = A4103Y = A4101Y × A4102Y

   Total Labor Compensation [BOC] = B4012Y = B4011Y + B4013Y
   B4011Y = Payroll = B801Y - B802Y - B803Y
   B4013Y = Fringe Benefits = B4014Y + B4015Y
   B801Y = Wage Earnings through December from MR23
   B802Y = Wage Earnings through December allocated Plant Construction from MR23
   B803Y = Wage Earnings through December allocated Plant Removal from MR23
   B4014Y = Benefits from FCC Statistics of Common Carriers
   B4015Y = Other Benefits = B4011Y × A4015Y/A4011Y
   A4015Y = Other Benefits [BS] = A4013Y - A4014Y
   A4011Y = Payroll [BS] from *BSSM*
   A4013Y = Fringe Benefits [BS] = A4012Y - A4011Y

* BS includes BOCs and LLs.

** Variable name designation used in the database. Prefix "A" denotes BS; "B" denotes BOC; "L" denotes LL. Suffix "Y" denotes annual; "Q" denotes quarterly; "M" denotes monthly.

A4014Y = Benefits [BS] from FCC Statistics of Common Carriers

Hours Worked [BOC] = B4101Y = B811Y × A4101Y/A811Y
B811Y = Employees from MR23
A811Y = Employees [BS] from MR23

Composition Index [BOC] = B4102Y = A4102Y (same as BS)
Total Labor Input [BOC] = Hours Worked × Composition Index
Total Labor Input [BOC] = B4103Y = B4101Y × B4102Y

Total Labor Compensation [LL] = L4012Y = L4011Y + L4013Y
L4011Y = Payroll [LL] = A4011Y - B4011Y
L4013Y = Fringe Benefits [LL] = L4014Y + L4015Y
L4014Y = Benefits from FCC Statistics of Common Carriers
L4015Y = Other Fringe Benefits [LL] = A4015Y - B4015Y
Hours Worked [LL] = L4101Y = L811Y × A4101Y/A811Y
L811Y = Employees [LL] = A811Y - B811Y
Composition Index [LL] = L4102Y = A4102Y same as BS
Total Labor Input [LL] = L4013Y = L4101Y × L4102Y

2. Develop Annual Labor Prices for BS, BOC, and LL by dividing the total labor compensation by total labor input [i.e., labor quantity].
   Labor Price [BS] = A4104Y = A4012Y/A4103Y
   Labor Price [BOC] = B4104Y = B4012Y/B4103Y
   Labor Price [LL] = L4104Y = L4012Y/L4103Y

3. Develop quarterly total labor compensation for BS, BOC, and LL. In general, the quarterly total labor compensation is calculated as:

   Quarterly Total Labor Compensation = quarterly payroll × (annual total labor compensation/annual payroll)

   Quarterly Total Labor Compensation [BS] = A4012Q = A4011Q × A4012Y/A4011Y
   A4011Q = Payroll [BS] = A801Q - A802Q - A803Q
   A801Q = Quarterly Wage Earnings from MR23
   A802Q = Quarterly Wage Earnings allocated to Plant Construction MR23
   A803Q = Quarterly Wage Earnings allocated to Plant Removal MR23

   Quarterly Total Labor Compensation [BOC] = B4012Q = B4011Q × B4012Y/B4011Y
   B4011Q = Payroll [BOC] = B801Q - B802Q - B803Q
   B801Q = Quarterly Wage Earnings MR23
   B802Q = Quarterly Wage Earnings allocated to Plant Construction MR23
   B803Q = Quarterly Wage Earnings allocated to Plant Removal MR23

   Quarterly Total Labor Compensation [LL] = L4012Q = L4011Q × L4012Y/L4011Y
   L4011Q = Payroll [LL] = A4011Q - B4011Q

4. Develop quarterly labor prices by linear interpolation of the annual prices developed in Step 2 above for BS, BOC, and LL. Assume that the annual labor prices are midyear (that is, the labor price on July 1).

SR-FAD-000552
Issue 1, May 1987

Linear interpolation of the annual prices to obtain the quarterly prices can be seen from the following hypothetical example.

Given: 7/1/76 price P76 = 100
7/1/77 price P77 = 120
Find: Quarterly prices P4Q76, P1Q77, P2Q77, P3Q77

Change in price: dP = P77 - P76 = 20

Since the assumption is that interpolation is linear, the quarterly change in price is dPq = 1/4 × dP = 5.

Therefore:

P4Q76 = P76 + dPq = 105 [Price as of 10/1/76]
P1Q77 = P76 + 2 × dPq = 110 [Price as of 1/1/77]
P2Q77 = P76 + 3 × dPq = 115 [Price as of 4/1/77]
P3Q77 = P76 + 4 × dPq = 120 [Price as of 7/1/77]

Quarterly Price of Labor [BS] = A4104Q = Quarterly Interpolation of A4104Y
Quarterly Price of Labor [BOC] = B4104Q = Quarterly Interpolation of B4104Y
Quarterly Price of Labor [LL] = L4104Q = Quarterly Interpolation of L4104Y

5. The quarterly labor input (labor quantity) is developed by dividing the quarterly total labor compensation (from Step 3 above) by the quarterly labor prices (from Step 4 above) for the BS, BOC, and LL separately.

   Quarterly Total Labor Input [BS] = A4103Q = A4012Q/A4104Q
   Quarterly Total Labor Input [BOC] = B4103Q = B4012Q/B4104Q
   Quarterly Total Labor Input [LL] = L4103Q = L4012Q/L4104Q

6. The labor input variable has been disaggregated into two components for use in the development of the cost model. Specifically, a quarterly price of labor (Step 4), and a quarterly labor quantity (Step 5) were developed.

4.4.3 Materials Variables

As with labor, analysis of the cost function requires development of two key materials variables: one that measures real materials input, and another that measures the price of materials. These were developed in four steps.

In the first step, the price of materials in each quarter was assumed to be proportional to the GNP (Gross National Product) price deflator.

In the second step, the total labor and materials expense was calculated as total operating expense net of certain capital-related costs:

Total Labor and Materials Expense = Total Operating Expense − Operating Rents − Total Depreciation and Amortization Expense

NOTE: The data required in this step are included in Monthly Report No. 5. Total operating expenses appear on line 50; operating rents appear on line 42, and depreciation and amortization appear on line 13.

In the third step, the materials expense was calculated as the amount by which the total labor and materials expense exceeds labor compensation:

Total Materials Expense = Total Labor and Materials Expense − Total Compensation.

Finally, real materials input was calculated by dividing the materials expense by the price index:

Real Materials Input = Total Materials Expense / Materials Price.

As with the labor variables, detailed descriptions are provided for the materials variables in Subsection 4.4.4 below. This information can be omitted with no loss in continuity.

4.4.4 Detailed Procedures Used to Develop Material Quantity and Material Price Indexes

Use the following steps to develop material quantity and material price indexes:

1. Develop the quarterly materials price index. This price index was assumed to be uniform for the BS and proportional to the GNP deflator.

   Price of Materials [BS] = A442Q from Bureau of Economic Analysis (BEA)

2. Develop the total labor and material expense on a quarterly basis for BS and BOC. This variable is calculated by subtracting the operating rents and the depreciation and amortization expenses from the total operating expenses.

   Total Labor and Materials Expense [BS] = A440Q from MR5
   Total Labor and Materials Expense [BOC] = B440Q from MR5

3. The total materials expense on a quarterly basis is developed by subtracting the total quarterly labor compensation [A4012Q for BS and B4012Q for BOCs] from Step 2.

   Total Materials Expense [BS] = A441Q = A440Q - A4012Q
   Total Materials Expense [BOC] = B441Q = B440Q - B4012Q
   Total Materials Expense [LL] = L441Q = A441Q - B441Q

4. The real materials input (i.e., material quantity) is developed by dividing the materials expense from Step 3 by the material price index from Step 1.

   Quantity of Materials [BS] = A445Q = A441Q/A442Q
   Quantity of Materials [BOC] = B445Q = B441Q/A442Q
   Quantity of Materials [LL] = L445Q = L441Q/A442Q

5. The materials input variable has been disaggregated into two components for use in the development of the cost model. Specifically, a quarterly materials price index (Step 1), and a quantity of materials (Step 4) were developed.

4.4.5 Capital Variables

The capital variables were developed following the methodology used by Christensen and Jorgenson (1969) for constructing capital accounts. These involved estimating the cost of capital, depreciation, re-evaluation, and taxation of several classes of capital goods.

Some capital items are rented by the former Bell System. The rental payments must be separated into price and quantity components to combine owned and rented capital into an index of total capital input.

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Owned Capital Stocks. The first step was to compute perpetual inventory estimates of the stock of each type of capital used in the former Bell System. In each quarter, the stock of each type of capital is the sum of stocks remaining from past investments of each vintage. Under the assumption that efficiency of capital goods declines geometrically, the replacement rate, δ, is a constant. Capital stock at the end of every quarter can be estimated from investment during the quarter and capital stock at the end of the previous year:

$$K_t^A = I_t + (1-\delta)K_{t-1}^A,$$

where K_t^A is end of quarter capital stock and I_t is the quantity of investment. To estimate these stocks, a benchmark estimate of capital stock investment in constant prices and a rate of replacement is required. Estimates of capital stocks for each of the 20 capital types are given in Table 4-1.

All investment data is taken from the Quarterly Report No. 2A, *Analysis of Changes in Telephone Plant Accounts*. This report gives the book value of investments placed in service each year from 1972 to 1982.

The next step was to deflate the book value of investment by an appropriate price deflator to estimate real investment. Price indexes for each capital type were taken from *Bell System Telephone Plant Indexes*. These yearly price indexes were interpolated to quarterly values.

Since the book value deflator should reflect the price at which these investments were contracted rather than the price prevailing when the asset was put in service, the book value of investment was deflated by the price index lagged by the average interval between the date at which the construction contract price is set and the date the plant is placed in service. Table 4-1 specifies the length of the lag for each capital type.

The current value of the real investment is calculated by using the current quarter price. This current value differs from the book value of investment by the revaluation which has taken place between the contract date and the current quarter price. This price index is also used to value the capital stock.

The depreciation rates used are taken from Christensen, Christensen, and Schoech (1981), and are reported in Table 4-1. The benchmarks used are also based on Christensen, Christensen, and Schoech data. The 1972 values of former Bell System capital stock reported in their paper are distributed between the BOCs and Long Lines (based on their respective percentages of book value gross stock reported in Quarterly Report 2A for December, 1972).

Capital Input from Owned Capital. To construct a quantity index of capital input, relative shares of capital service flows were required. In the absence of taxation, the value of capital services for a particular asset is the sum of the cost of capital and depreciation, less revaluation:

$$\text{Formula 1:} \quad p_{Kt}K_t = [p_{A,t-1}\, r_t + p_{At}\,\delta - (p_{At} - p_{A,t-1})]K_{t-1}^A$$

Given the quantity of each type of asset held, K_{t-1}^A, the acquisition price, p_{At}, and the rate of replacement, δ, only the cost of capital, r_t, is required to compute capital services for each type of asset. The embedded cost of capital used for capital budget planning in the former Bell System was used for r_t.

This formula is appropriate for a single class of assets. For several classes of assets, property compensation is the sum of price times quantity of capital services for all classes of assets. It is assumed that the cost of capital is the same for all assets held by the former Bell System.

Formula 1 holds only in the absence of taxation. Christensen and Jorgenson (1969) show that the appropriate formula in the presence of income and property taxes is:

$$\text{Formula 2:}\quad p_{K,t}K_t = \frac{1 - u_t z_t - k_t + y_t}{1 - u_t}\,[p_{A,t-1}r_t + p_{A,t}\delta$$

$$- (p_{A,t} - p_{A,t-1})]\,K^A_{t-1} + p_{A,t-1}\tau_t K^A_{t-1}.$$

where u_t is the effective corporate profits tax rate

z_t is the present value of imputed depreciation allowances on one dollar's worth of investment

k_t is the investment tax credit, different from zero for some years for all capital types except buildings

y_t is $k_t u_t z_t$ in 1962 and 1963 and is set equal to zero for all other years; it is used to account for the fact that the investment tax credit was deducted from the value of an asset for depreciation in those years

τ_t is the property tax rate.

The former Bell System effective corporate profits tax rate is estimated as the ratio of income taxes paid plus the investment tax credit to property compensation less property taxes and the imputed value of depreciation allowances for tax purposes.

Income taxes paid are federal plus state and local income taxes. The *BSSM* gives federal income taxes, and state and local income taxes, as well as the amount of the investment tax credit.

Imputed depreciation allowances depend on depreciation formulas allowed for tax purposes, the lifetimes of assets used for calculating tax depreciation and the opportunity cost of capital. It was assumed that the appropriate opportunity cost for the discounting future depreciation allowances is the forward-looking former Bell System cost of capital.

Table 4-1. Asset Classes for Capital Input

Capital Type	Lag Between Contract and Placement of Investment	Annual Depreciation Rate
Buildings	4 quarters	.0270
COE: Manual	2 quarters	.1621
Panel	3 quarters	.1850
Step-by-Step	2 quarters	.1042
Crossbar	3 quarters	.0539
Circuit	2 quarters	.0735
Radio	2 quarters	.0833
Electronic	3 quarters	.0310
Station Apparatus	none	.1144
Station Connections	none	.1337
Large PBX	1 quarter	.1400
Pole Lines	none	.0657
Aerial Cable	none	.0463
Underground Cable	none	.0284
Buried Cable	none	.0383
Submarine Cable	none	.0657
Aerial Wire	none	.1645
Underground Conduit	none	.0256
Furniture and Office Equipment	none	.0822
Vehicles and Other Work Equipment	none	.1802

For 1972 to 1980, *Engineering Economy* was used to specify depreciation formulas. This specifies a 1.5 declining balance formula for structure, while for all other capital it specifies double declining balance switching to a sum of years digit formula at the optimal point. Lives used are those specified in *Depreciation Guide* published by The Commerce Clearing House. Beginning in 1981, the lifetimes and formulas allowed by the ACRS depreciation system were used.

The property tax rate is the ratio of all operating taxes related to capital input to the value of the total capital stock at the beginning of the period. Operating taxes related to capital input include property taxes, and the category other taxes. Both are reported in the *BSSM*.

It was assumed that the real flow of capital services from each type of asset is proportional to the stock of the asset at the end of the previous period:

$$K_{it} = q_{Ki} K^{A}_{it-1} .$$

The price per unit of capital service can be obtained by dividing the service flow by the lagged stock and normalizing to unity in the base year.

Capital Input from Rented Capital. Expenditures for rented capital are contained in the Monthly Report No. 5. The expenditures represent the value of capital services from rented capital goods. The bulk of these capital goods are structures. It is necessary to separate the expenditures into price and quantity components. The CPI rent index was used to deflate rental expenditures.

Aggregate Bell System Capital Input. The quantity indexes of capital stock and capital input were computed as Tornqvist indexes of the quantity indexes of owned and rented capital. Dividing the total value of capital stock and capital services by the quantity indexes yields the price indexes of capital stock and capital services.

Capital input for each type of capital is proportional to the corresponding stock. However, as a result of changes in the composition of the aggregate capital stock, aggregate capital input is not proportional to aggregate capital stock. Therefore, it is incorrect to use aggregate capital stock to represent aggregate capital input.

A more detailed description of the development of the capital input variables is presented in Subsection 4.4.6. This subsection, as with the previous input variable descriptions, can be omitted without loss in continuity. Subsection 4.4.6 is provided for those who require a detailed, step-by-step description of how the capital variables were constructed for this model.

4.4.6 Detailed Procedures Used to Develop Price and Quantity Indexes for the Capital Variable

Objective: To develop price and quantity indexes for total capital input that reflect the value of owned capital stock as well as the value of rented capital.

Use the following steps:

1. Develop the booked investment by class of plant on a quarterly basis for the BS and LL. There are 26 classes of plant considered. For example, the quarterly Building Booked Investment (A06Q) is calculated from the change in the year-to-date Building Booked Investment. (A06Y) is calculated from the Quarterly 2A Report.

Class of Plant	Variable Name	Remarks
Building	A06Q or L06Q	Prefix A = BS; L = LL. Same remark for all classes of Plant
COE: Manual	A07Q	
COE: Panel	A08Q	
COE: Step-by-Step	A09Q	
COE: Crossbar	A10Q	
COE: Circuit	A11Q	
COE: Radio	A12Q	
COE: Electronic	A13Q	
COE: Nondedicated Circuit	A14Q	
COE: DDS Circuit	A15Q	
COE: Pair Gain System	A16Q	
COE: Other Circuit Equipment	A17Q	
COE: Small Value Items	A18Q	
COE: Total	A19Q	
Total Station Apparatus	A20Q	
Total Station Connections	A21Q	
Total PBX	A22Q	
Pole Lines	A23Q	
Underground Conduit	A24Q	
Total Aerial Cable	A25Q	
Total Underground Cable	A26Q	
Buried Cable	A27Q	
Submarine Cable	A28Q	
Total Aerial Wire	A29Q	
Total Furniture and Office Eqpt	A30Q	
Total Vehicles and Other	A31Q	

2. The booked investment for the BOCs was developed by class of plant (quarterly) by subtracting the Long Lines booked investment (L prefix) identified in Step 1 from the total former Bell System (A prefix) booked investment also identified in Step 1.

   Example: Building Book Investment for Quarter [BOC] = B06Q = A06Q - L06Q

3. Develop a real investment for each class of plant. This step adjusts the booked investment to account for the lag occurring between the contract and the actual placement of the investment. This intermediate step is required before developing the current value of the investment at the time of placement. To arrive at a current value of investment, first develop the real investment at contract by dividing the booked investment by the corresponding Telephone Price Index (TPI) appropriately lagged to reflect the time required to place the investment in service. The real investment can be thought of as being the quantity of investment purchased, since it is developed by dividing the total investment cost by a price of investment (the TPI). Once this real investment, or investment quantity, is calculated, the current dollar investment at the time the investment was placed in service can be developed by multiplying the real investment by the TPI in effect at the time of placement. To clarify this concept, consider the following example:

   Assume an investment, I=$100, was contracted for on 1/1/75 and placed in service and on the books on 1/1/76. The book value, BV, is therefore $100. The objective is to identify the current value of the investment on 1/1/76 since the $100 book value represents the actual amount paid at time of contract (1/1/75). Therefore:

$$\text{Real Investment RI} = \frac{BV}{TPI[1/1/75]} = \frac{100}{.80} = \$125$$

Assuming TPI[1/1/75] = .80, and

Current Dollar Investment = CI = RI × TPI [1/1/76] = $118.75
as of 1/1/76

Assuming TPI [1/1/76] = .95.

Therefore, the $100 investment as of 1/1/75 had a current dollar value of $118.75 when placed in service on 1/1/76.

Development of Real Investment

a) Develop price index by class of plant on quarterly basis.
Buildings: Price Index [BS] = A115Q = Linear interpolation of A115Y

- See Labor Variable for explanation of Linear interpolation.
- A115Q, A125Q, A135Q...A185Q, A225Q, A235Q...A335Q are the price indexes by class of plant.
- BS Price Indexes used for BS, BOC, and LL.

b) Develop Real Investment using price index appropriately lagged by class of plant.
Buildings: Real Investment [BS] = A111Q = A06Q/A115Q(-4)

- A111Q, A121Q, A131Q...A181Q, A221Q, A231Q...A331Q are the real investments by class of plant for BS.
- Similar series for Long Lines ("L" prefix instead of "A").

4. Develop Quarterly Current Dollar investment by class of plant.
Buildings: Current Dollar Investment = A117Q = A111Q × A115Q

- A117Q, A127Q,...A187Q, A227Q,...A337Q are current dollar investment by class of plant for BS.
- Similar series for Long Lines ("L" prefix replaces "A").

5. Next, determine the value of the owned capital stock on a quarterly basis for each class of plant. This is obtained using the formula:

$$K_t^A = I_t + (1-d) \times K_{t-1}^A. \qquad \text{Equation (1)}$$

which was discussed earlier in this section. The value of the capital stock, also called constant dollar stock, can be interpreted as the cost of purchasing the assets owned by the company.

Buildings: Constant Dollar Stock [BS] = E114Q (Use Equation (1) with appropriate values for I_t, d, and benchmark stock value.)

- I_t = A111Q.
- d = depreciation rate on quarterly basis.
- benchmark = the net asset value of capital stock as of the fourth quarter of 1972. This was obtained from a 1981 study by Christensen Associates and is required as a starting point since Equation (1) relates the stock value in one period, K_t^A, with the value in the previous period, K_{t-1}^A. The benchmark was discussed earlier in this section.
- E114Q, E124Q,...E184Q, E224Q...E334Q is the constant dollar stock by class of service for BS.
- Similarly, replacing prefix "E" with "F" identifies the constant dollar stock by class of plant for LL.

6. Once the constant dollar stock value is developed, the next step is to develop the current dollar stock value for each quarter by class of plant. This is analogous to developing the current dollar investments from the real investments described in Steps 3 and 4.

Buildings: Current Dollar Stock [BS] = E112Q = A115Q × E114Q

- E112Q, E122Q,...E182Q, E222Q...E332Q is the current dollar stock by class of plant for BS.
- Similarly, replacing "E" with "F" identifies the current dollar stock for LL by class of plant.

7. The constant dollar stock and the current dollar stock by class of plant is used to develop a value of total capital stock disaggregated into price and quantity. The total value of capital stock is obtained by adding the current dollar stock for each class of plant in each quarter. The quantity of capital stock is developed by using the Tornqvist indexing procedure. The price then can be calculated since the total value of capital stock is equal to the product of the quantity of stock and the price of the stock. The general form of a Tornqvist index was described earlier in the data section and is shown below:

$$\ln(X_t/X_{t-1}) = \sum_{i}^{\substack{\text{Class}\\\text{of}\\\text{Plant}}} W_{i,t}^{o} \ \ln(X_{i,t}/X_{i,t-1}). \qquad \text{Equation (2a)}$$

where

$$W_{i,t} = (W_{i,t} + W_{i,t-1})/2 \text{ , and} \qquad \text{Equation (2b)}$$

$$W_{i,t} = P_{i,t}\, X_{i,t} / \sum_{j}^{\text{Class of Plant}} P^{o}_{j,t}\, X_{j,t} \qquad \text{Equation (2c)}$$

and

X_t = the aggregate quantity index.

Applying equations (2) to the capital stock variable:

$X_{i,t}$ = constant dollar stock by class of plant E114Q...[BS], and F114Q...[LL]
$P_{i,t}$ = price index by class of plant A115Q...[BS and LL].

Using Equations (2a), (2b), and (2c), the quantity ratios, X_t/X_{t-1}, are calculated for all the quarters in the study period. As indicated in the description of the Tornqvist index, an arbitrary value must be assigned to the index at some base period to calculate the quantity index for the remaining periods. In the case of the capital stock variable, the base period chosen was the second quarter of 1977. The quantity index, X_t, for the second quarter of 1977 is the sum of the current dollar stock for all classes of plant. Choosing this value makes the price index equal 1.0 for the base period (the second quarter of 1977).

Quantity Index of Total Capital Stock = E344Q[BS], or F344Q[LL]

The quarterly value of total capital stock is obtained by summing the quarterly current dollar stock for all classes of plant.
Total Capital Stock = E342Q[BS] or F342Q[LL]

The corresponding price index is calculated by the following formula:

Price Index of Total Capital Stock = E341Q = E342Q/E344Q
(Replacing "E" with "F" will give the LL formula).

8. The first seven steps were concerned with developing the value of the capital stock owned by the company. This value can be thought of as the replacement value of the property owned by the company. The next step in the process addresses the problem of quantifying the rental value, or the service flow, of the owned capital. As explained earlier, the formula used to calculate service flows for a class of plant in the presence of income and property taxes was developed by Christensen and Jorgenson (1969) and is given by:

$$\text{Service flow for a class of plant} = P_{kt} \times K_t = \frac{1-u_t z_t-k_t+y_t}{1-u_t}\,[P_{A,t-1} \times r_t + p_{A,t} \times d \qquad \text{Equation (3)}$$

$$-(p_{A,t} - p_{A,t-1})]K^{A}_{t-1} + P_{A,t-1} \times \tau_t \times K^{A}_{t-1}$$

where

K^{A}_{t-1} = quantity of each type of asset held
P_{At} = acquisition price
d = rate of replacement
u_t = effective corporate profits tax rate
z_t = present value of imputed depreciation allowances on one dollar's worth of investment
k_t = investment tax credit, different from zero for some years for all types of capital except buildings
y_t = 0 for all years except 1962, 1963; variable accounts for the fact that the investment tax credit was deducted from the value of an asset for depreciation in 1962 and 1963
τ_t = property tax rate
r_t = cost of capital.

To calculate the service flow by class of plant using the Equation (3), intermediate steps are required to calculate all the necessary variables.

a) The cost of capital [r_t] was assumed to be the same for all assets held by the former Bell System.
Cost of capital = A528Q from former Bell System historical cost of capital.

b) Present Value of Depreciation [z] was discussed earlier in this paper. From 1972 to 1980, an accelerated depreciation structure was used as specified in *Engineering Economy*. Beginning in 1981, the lives and formulas allowed by the ACRS depreciation system were used.

Buildings: Present Value of Depreciation [BS] Z = A541Q = A537Q × A541CQ + (1-A537Q) × A541BQ

A537Q = Variable used to choose correct depreciation formula by year; 1 = years after 1980, 0 = prior to 1981
A541BQ = Present Value Accelerated Depreciation: Buildings prior to 1981
A541BQ = [(2+A528Q)/2] × [D/(1+A528Q)]/[(1-(1-D/T)/(1+A528Q))×T]
D = Accelerated Depreciation Ratio
T = Tax Life: Buildings = A430Q
A528Q = Cost of Capital
A541CQ = Present Value of ACRS Depreciation: Buildings after 1980
= [(1-D/T)/(1+A528Q)]**TS+[(D/(D+A528Q×T))×(1-(1/1+A528Q)**TS)×(1-D/T)**TS]
TS = Variable represents optimal year to switch from declining balance to straight line depreciation. Value maximizes present value of z

Variables for Remaining Classes of Plant:

Class of Plant	z	PV of Accel. Dep.	PV of ACRS Dep.
COE: Manual	A542Q	A542BQ	A542CQ
COE: Panel	A543Q	A543BQ	A543CQ
COE: Step-by-Step	A544Q	A544BQ	A544CQ
COE: Crossbar	A545Q	A545BQ	A545CQ
COE: Circuit	A546Q	A546BQ	A546CQ
COE: Radio	A547Q	A547BQ	A547CQ
COE: Electronic	A548Q	A548BQ	A548CQ
Station Apparatus	A549Q	A549BQ	A549CQ
Station Connections	A550Q	A550BQ	A550CQ
PBX	A551Q	A551BQ	A551CQ
Pole Lines	A554Q	A554BQ	A554CQ
Aerial Cable	A555Q	A555BQ	A555CQ
Underground Cable	A556Q	A556BQ	A556CQ
Buried Cable	A557Q	A557BQ	A557CQ
Submarine Cable	A558Q	A558BQ	A558CQ
Aerial Wire	A559Q	A559BQ	A559CQ
Underground Conduit	A560Q	A560BQ	A560CQ
Furniture & Office	A561Q	A561BQ	A561CQ
Vehicles & Other	A562Q	A562BQ	A562CQ

Tax Lives by Class of Plant:

Class of Plant	Tax Life
COE	A431Q
Station Apparatus	A432Q
Station Connections	A433Q
PBX	A434Q
Pole Lines	A435Q
Cable & Wire	A436Q
Underground Conduit	A437Q
Furniture & Office	A438Q
Vehicles & Other	A439Q

c) Investment Tax Credit [k]

k = A469Q = A413Y/[(A341Q - A117Q) for the year]

A413Y = Investment Tax Credit [BS] from *BSSM*
A341Q = Total Current Dollar Investment = sum of A117Q. A127Q...A337Q
A117Q = Current Dollar Investment: Buildings

d) Effective Corporate Profits Tax Rate [u_t]

u_t = A535Q = A469Q(A528Q(E3420(-1)-E112Q(-1))+E347BQ-E117BQ-E348BQ +E118BQ)+A463y)/(A531Q-A465Y-A593Q)

A469Q = Investment Tax Credit
A528Q = Cost of Capital
E342Q(-1) = Value of Total Capital Stock
E112Q(-1) = Current Dollar Stock: Buildings

E347BQ = Estimated Depreciation = Sum of estimated depreciation by class of plant E117BQ. E127BQ...E337BQ

For Buildings: E117BQ = EXP(GR115Y)×E112Q(-1).0270

GR115Y = Asset Price Growth Rates for Building
GR115Y = [log(A115Y/A115L)]/5
A115Y = TPI Buildings no lag
A115L = TPI Buildings lagged by 5 periods
.0270 = Annual depreciation rate for Buildings

E348BQ = Estimated total Capital Gains=sum of estimated capital gains by class of plant E118BQ.E128BQ,...,E338BQ

For Buildings: E118BQ = [EXP(GR115Y)-1]E112Q(-1)

A463Y = Income Tax = A411Y+A418Y
A411Y = Federal Tax from *BSSM*
A418Y = State Tax from *BSSM*
A531Q = Capital Input = A528Q×E342Q(-1)+E347BQ-E348BQ+A465Y+A463Y
A465Y = Capital Taxes = A414Y+A416Y
A414Y = Property Taxes from *BSSM*
A416Y = Other Taxes from *BSSM*

A593Q = Total depreciation = sum of depreciation by class of plant A571Q,A572Q. ... A592Q

For Buildings: A571Q = A541Q(A528Q×E112Q(-1)+E117BQ-E118BQ)
A541Q = Present Value of Depreciation [z]
A528Q = Cost of Capital
E112Q(-1) = Current Dollar Stock: Buildings
E117BQ = Estimated Depreciation for Buildings
E118BQ = Estimated Capital Gains for Buildings

e) Property Tax Rate (τ)

τ = A500Q = A465Y/(E342Q at end of previous year)
A465Y = Capital Taxes
E342Q = Value of Total Capital Stock [replacement value]

f) Depreciation rate on a quarterly basis (Step d) by class of plant is tabulated earlier in this section.

9. Using the data gathered in Step 8 (above), the Service Flows by class of plant can be calculated. The algorithm presented next is used to calculate the service flows for all classes of plant with a minor modification for Buildings. In the case of Buildings, the investment tax credit (given by A469Q in the numerator) is set to zero.

For COE Manual: Service Flows [BS] = A602Q = ((1 - A469Q - A535Q × A542Q/(1 - A535Q))×
(A528Q×E122Q(-1)+E127BQ-E128BQ)+A502Q
A502Q = Property Taxes = A500Q × E122Q (-1).

where

A500Q = Property Tax Rate

E122Q(-1) = Current Dollar Stock [COE Manual] lagged by one quarter

E128BQ=$(P_{A,t}-P_{A,t-1})K_{t-1}^{A}$ term in Equation (3)=Estimated Capital Gains

E127BQ=$P_A K_{t-1}^{A} d$ term in Equation (3)=Estimated Depreciation

A528Q=Cost of Capital (r)

E122Q(-1)=$P_{A,t-1}K_{t-1}^{A}$ term in Equation (3)=Current Dollar Stock lagged by one quarter.

A542Q=z term in Equation (3)=Present Value of Depreciation for *COE Manual*

A535Q=u_t term in Equation (3)=Income Tax Rate.

The Service Flow and associated variables by Class of Plant are given by:

Class of Plant	Svc Flow	Prop. Tax	Estimated Cap. Gain	Estimated Depreciation	PV Depreciation
Buildings	A601Q	A501Q	E118BQ	E117BQ	A541BQ
COE: Manual	A602Q	A502Q	E128BQ	E127BQ	A542BQ
COE: Panel	A603Q	A503Q	E138BQ	E137BQ	A543BQ
COE: Step-by-Step	A604Q	A504Q	E148BQ	E147BQ	A544BQ
COE: Crossbar	A605Q	A505Q	E158BQ	E157BQ	A545BQ
COE: Circuit	A606Q	A506Q	E168BQ	E167BQ	E546BQ
COE: Radio	A607Q	A507Q	E178BQ	E177BQ	A547BQ
COE: Electronic	A608Q	A508Q	E188BQ	E187BQ	A548BQ
Station Apparatus	A609Q	A509Q	E228BQ	E227BQ	A549BQ
Station Connections	A610Q	A510Q	E238BQ	E237BQ	A550BQ
Large PBX	A611Q	A511Q	E248BQ	E247BQ	A551BQ
Pole Lines	A614Q	A514Q	E258BQ	E257BQ	A554BQ
Aerial Cable	A615Q	A515Q	E268BQ	E267BQ	A555BQ
Underground Cable	A616Q	A516Q	E278BQ	E277BQ	A556BQ
Buried Cable	A617Q	A517Q	E288BQ	E287BQ	A557BQ
Submarine Cable	A618Q	A518Q	E298BQ	E297BQ	A558BQ
Aerial Wire	A619Q	A519Q	E308BQ	E307BQ	A559BQ
Underground Conduit	A620Q	A520Q	E318BQ	E317BQ	A560BQ
Furniture & Office Eqpt.	A621Q	A521Q	E328BQ	E327BQ	A561BQ
Vehicles & Other Eqpt.	A622Q	A522Q	E338BQ	E337BQ	A562BQ

A similar analysis is performed to determine the Service Flows for Long Lines [LL]. The LL variables are the same as shown above with "A" replaced by "L" and "E" replaced by "F."

10. Once the Service Flows have been calculated for the former Bell System [BS] and Long Lines [LL], the BOC Service Flows can be calculated by subtracting the LL Service Flow from the BS Service Flow. Performing this calculation results in a set of BOC variables B601Q, B602Q...B622Q which identify the Service Flows by the classes of plant shown above.

11. The Services Prices by class of plant are calculated from the Service Flows developed above. The following general formula is used to calculate the service prices:

Service Price = Service Flow/Constant Dollar Stock at beginning of quarter.

Recall, from the development of the constant dollar stock, that this variable can be thought of as the quantity of capital stock. The Service Price is a measure of the cost the company would incur if it were to rent the owned capital:

For Buildings: Service Price [BS] = A631Q = A601Q/E114Q(-1).

Similar equations can be written for the remaining classes of plant (A632Q, A633Q, ... A652Q).

12. The quarterly Service Prices, by class of plant, are used together with the constant dollar stock values at the beginning of the quarter, K_{t-1}^{A}, to generate a quantity index, k_t, of owned capital input. To do this, use the Tornqvist Index procedure discussed earlier in this document.

To evaluate the quantities, k_t, from the Tornqvist ratios of K_t/K_{t-1}, a value is assigned to K_t in any period. Once a value of K_t is known in the base period, the K_t in the remaining quarters can be calculated. In our case, K_t for the second quarter of 1977 was set equal to the sum total of the Service Flows by class of plant for that period. This choice for the quantity index forces the corresponding price index to be united in the base period:

Quantity of Owned Capital Input [K_t] = A655Q
A655Q = Tornqvist Index of:

[K_{t-1}^{A}], E114Q(-1) ... E334(-1) with A631Q, ... A652Q as prices.

For the base period, second quarter of 1977, set k_t equal to sum of Service Flows for each class of plant, i.e., sum of A601Q, A602Q ... A611Q, A614Q ... A622Q.

13. The Price Index of Owned Capital is calculated by dividing the sum of all class of plant Service Flows by the quantity of owned capital input calculated in Step 12:

Price of Owned Capital Input = A656Q

$$A656Q = \frac{\text{Sum A601Q ... A611Q, A614Q ... A622Q}}{\text{A655Q}}.$$

14. To develop price and quantity indexes of input, capital consideration must be given to rented capital as well as to owned capital. Therefore, calculate price and quantity indexes for building rents. These indexes will be used in conjunction with the owned capital indexes to develop composite capital input price and quantity indexes:

    Value of Building Rents = A658Q from MR5 Rental Expense
    Quantity of Building Rent = A659Q = A658Q/A631Q
    where A631Q=Service Price Buildings.

15. The composite or total capital input quantity and price indexes are developed using a Tornqvist Index of owned capital input quantity (A655Q) and rented building quantity (A659Q) with price of owned capital input (A656Q) and price of rented buildings (A631Q) as prices. As with the other Tornqvist Indexes, the second quarter of 1977 was chosen as the base period, and the quantity index was set equal to the sum total of Service Flows by class of plant (A601Q ... A622Q) and the value of building rents (A658Q):

$$\text{Quantity of Total Capital Input} = \text{A661Q}$$
$$\text{Price of Total Capital Input} = \text{A660Q} = \frac{\text{A601Q} + \ldots \text{A622Q}}{\text{A661Q}}.$$

16. The BOC Service Prices, Quantity and Price of Owned Input Capital, and Quantity and Price of Total Input Capital are developed in the same manner as for the BS. A summary of the variables is presented below:

    a) Service by class of plant B631Q ... B652Q: Divide the Service Flows (B601Q ... B622Q) by the constant dollar stock at the beginning of the period (G114Q(-1) ... G334Q(-1)). The BOC constant dollar stock was developed by subtracting the LL value from the BS value (e.g., for Buildings B601Q = E114Q - F114Q).

    b) Quantity of Owned Capital Input = B655Q

    c) Price of Owned Capital Input = B656Q

    d) Value of Building Rents = B658Q

    e) Quantity of Building Rents = B659Q

    f) Price of Building Rents = B631Q

    g) Price of Total Capital Input = B660Q

    h) Quantity of Total Capital Input = B661Q.

The equivalent Long Lines variables can be identified by replacing "B" with "L" and "G" with "F" in the items shown above.

4.4.7 The Output Variable

The key output variable is real output. This variable was developed in three steps. In the first step, monthly real output of each type of service, i, was calculated by the appropriate price indexes:

$$\text{Monthly Real Output}_i = \text{Monthly Revenues}_i \,/\, \text{Monthly Price}_i.$$

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Monthly revenues for each service are available in Monthly Report No. 4. Price indexes are available from the Bureau of Labor Statistics. Table 4-2 shows the categories of output and the related BLS deflator.

In the second step, quarterly price indexes for each service were estimated by dividing quarterly revenues by quarterly real output. The quarterly revenues and real output figures are derived from the respective monthly figures.

Finally, the services are aggregated to a single output using Tornqvist indexing procedures.

4.4.8 Detailed Procedures to Develop the Output Variable

1. Calculate the real monthly output for each type of service by dividing the monthly revenue by the appropriate price indexes.

Monthly Revenues by Service from MR4 Report.

Service	Monthly Revenue*
Location Service-IS	A701M
Location Service-IA	A702M
MTS-IS	A703M
MTS-IA	A704M
WATS-IS	A705M
WATS-IA	A706M
Private Ln-IS	A707M
Private Ln-IA	A708M
Other Toll-IS	A709M
Other Toll-IA	A710M
Miscellaneous	A711M
Total Revenue before Uncollected-IS	A712M
Total Revenue before Uncollected-IA	A713M

Monthly Service Price Indexes from the Bureau of Labor Statistics.

*A = BS. Replace "A" with "L" to obtain Long Lines variables.

Service	Price Index
Local Service	A721M
MTS-IS	A723M
MTS-IA	A724M
WATS-IS	A725M
WATS-IA	A726M
PL-IS	A727M
Directory Advertising	A731M*

* Used to develop real output for miscellaneous services.

Table 4-2. Output Categories and BLS Price Index References

Category		BLS Reference
Local	- Intrastate	4811-1
	Interstate	4811-1
MTS	- Intrastate	4811-211
	Interstate	4811-212
WATS	- Intrastate	4811-214-12
	Interstate	4811-214-11
Private Line	- Intrastate	4811-311
	Interstate	4811-311
Other Toll	- Intrastate	4811-2
	Interstate	4811-2
Miscellaneous		4811-911

Monthly Real Output Local Service-IS = A701M/A721M similarly for remaining services.

2. The quarterly revenues were developed by service by accumulating the monthly revenues to the quarter. Quarterly Revenues by Service = A701Q, A702Q ... A711Q [Replace "A" with "L" to obtain Long Lines].

3. The real output (quantities) are developed by quarter accumulating the monthly real output to the quarter for each service.

   Quarterly Quantities by Service = A741Q ... A751Q ["A" replaced by "L" for Long Lines].

4. Once the quarterly quantities and revenues are calculated, the quarterly price indexes can be obtained.

   Price Index by Service = A721Q ... A731Q

   $$A721Q \ldots A731Q = \frac{A701Q}{A741Q} \ldots \frac{A711Q}{A751Q}$$

   [Replace "A" with "L" to obtain Long Lines.]

5. The aggregate output is calculated using Tornqvist Indexing.

   Aggregate Output = A770Q

   A770Q = Tornqvist Index of service quantities A741Q ... A751Q with Service Prices A721Q ... A731Q.

   Second quarter of 1977 was selected as the base period. The value of Aggregate Output was set equal to total service revenues for that period. This, together with the relationships from the Tornqvist Index, enables us to evaluate the aggregate output for each quarter.

   Total Service Revenues = sum of A701Q through A711Q

6. The Price Index for the Aggregate Output is determined by dividing the Total Service Revenues for each quarter by the corresponding Aggregate Output.

   Price Index of Aggregate Output = A772Q

   $$A772Q = \frac{A701Q + \ldots + A711Q}{A770Q}$$

   [Replace "A" with "L" to obtain Long Lines.]

7. In a similar fashion, the output quantity and corresponding price was developed for subtotal categories of services. Specifically,

Subtotal Category	Individual Service Revenues Included
Local	Local IS, Local IA, & Misc
Toll	MTS-IS, MTS-IA, Other Toll-IS, Other Toll -IA
IS Toll	MTS-IS, WATS-IS, PL-IS, Other Toll-IS
Non-IS Toll	Local IS, Local IA, MTS-IA, WATS-IA, PL-IA Other Toll-IA & Misc.

Subtotal Category	Quantity	Price
Local	A775Q	A776Q
Toll	A780Q	A782Q
IS Toll	A785Q	A786Q
Non-IS Toll	A790Q	A791Q

8. The BOC output variables are developed by taking the difference of BS variables and LL variables. The following procedure was used:

   a. Calculate quarterly BOC service revenues.
   Revenues by Service = B701Q ... B711Q
   B701Q ... B711Q = (A701Q - L701Q) ... (A711Q - L711Q)

   b. Calculate quarterly BOC service quantities.
   Quantity by Service = B741Q ... B751Q
   B741Q ... B751Q = (A741Q - L741Q) ... (A751Q - L751Q)

   c. Calculate quarterly price index by service.
   Price Indexes by Service = B721Q ... B731Q

   $$\text{B721Q} \ldots \text{B731Q} = \frac{\text{B701Q}}{\text{B741Q}} \ldots \frac{\text{B711Q}}{\text{B751Q}}$$

   d. The aggregate output and price are calculated using the Tornqvist Index.

   Aggregate Quantity = B770Q
   Aggregate Price = B772Q

   e. Develop subtotal service category prices and quantities.

Subtotal Category	Quantity	Price
Local	B775Q	B776Q
Toll	B780Q	B782Q
IS Toll	B785Q	B787Q
Non-IS Toll	B790Q	B791Q

4.4.9 The Network Variable

The network variable represents the size of the system as distinguished from its level of output. This distinction is necessary because the cost effects of increases in output depend on the extent to which the output increases are accompanied by network expansion. For example, an increase in output that occurs without an addition to network will be less costly than an identical increase that is accompanied by network expansion. In other words, the extent to which costs increase in response to an increase in output depends upon the extent to which the size of the network is also increasing.

The network variable is measured by the number of main equivalent telephones in service in the system.

4.4.10 Details on the Development of the Network Variable

The number of mains and equivalent mains in service at the end of each quarter are used as a measure of the size of the network used to provide service. The MR7 report is the basic source for these data.

Total Main & Equivalent Main - In Service End of Year = B905Y
Total Mains & Equivalent Mains - Net Gain = B903M

- Quarterly net gains are obtained by adding the appropriate monthly quantities (B903M).
- Quarterly net gains are added to the year end in service quantity to obtain the first second and third quarter quantities.

4.4.11 The Technology Variable

Annual technology index figures were calculated by taking weighted averages of annual former Bell System research and development expenditures (R&D) of the current and preceding twenty two years. [NOTE: Expenditures beyond a fifteen-year lag have virtually no effect on the results.] Quarterly values were interpolated from the annual figures.

The weights applied to past R&D expenditures were those of a Poisson distribution with a mean lag of six years. Specifically, the proxy for the level of technology in year t was calculated by:

$$\frac{\sum_{s}(\text{R\&D})/(\text{CPI})_{t-s} \cdot (e^{-6} \cdot 6^{s}/S!)}{\sum_{s}(e^{-6} \cdot 6^{2}/S!).}$$

where (R&D/CPI) represents real R&D expenditures as deflated by the consumer price index. This procedure follows that used by H. D. Vinod.[30]

30. H. D. Vinod, "Bell System Scale Economies and Estimation of Joint Production Functions," Bell Telephone Laboratories Inc., p. 14.

4.5 Data Sources

The following tables list the specific data sources that are required for the computations used in this Special Report. Full citations for each data source marked with an asterisk may be found in Section 8, References.

Figure 4-1. Labor Quantity and Price

	Item	Monthly Report	Years	Page or Schedule	Line	Col.
1.	Total Labor Compensation (A4012Y)	Bell System Statistical Manual*				
2.	Hours Worked (A4101Y)	Christensen, Christensen & Schoech*				
3.	Compensation Index (A4102Y & B4102Y)	Christensen, Christensen & Schoech*				
4.	Wage Earnings or Quarterly Wage Earnings (B801Y A801Q, & B801Q)	MR23	71 & 72	1	51	R or S
			73-76	3	4	F or D
			77-83	D	4	A
5.	Wage Earnings or Quarterly Wage Earnings Allocated to Plant Construction (B802Y, A802Q, & B802Q)	MR23	71 & 72	Not Available		
			73-76	3	7	F or D
			77-83	D	4	D
6.	Wage Earnings or Quarterly Wage Earnings Allocated to Plant Removal (B803Y, A803Q, & B803Q)	MR23	71 & 72	Not Available		
			73-76	3	8	F or D
			77-83	D	4	E
7.	Benefits (B4014Y, A4014Y, & L4014Y)	FCC Statistics of Communication Common Carriers*				
8.	Payroll (A4011Y)	Bell System Statistical Manual*				
9.	Employees (A811Y & B811Y)	MR23	71 & 72	1	17	C
			73-76	1	1	I
			77-83	A	6	A

4.5 Data Sources (continued)

Figure 4-2. Material Quantity and Price

	Item	Monthly Report	Years	Page or Schedule	Line	Col.
1.	Tot. Oper. Expenses (A440Q)	MR5	71-75	1	48	A
			76-79	1	49	A
			80-83	1	46	A
2.	Operating Rents (A440Q)	MR5	71-75	1	41	A
			76-79	1	42	A
			80-83	1	39	A
3.	Total Depreciation and Amortization Expense (A440Q)	MR5	71-83	1	13	A
4.	Total Earnings (A801Q)	MR23	71 & 72	1	51	S
			73-76	3	4	D
			77-83	D	1	A
5.	Earnings Allocated to Plant Construction (A802Q)	MR23	71 & 72	1	Not Available	
			73-76	3	7	D
			77-83	D	1	D
6.	Earnings Allocated to Plant Removal (A803Q)	MR23	71 & 72	1	Not Available	
			73-76	3	8	D
			77-83	D	1	E
7.	Payroll (A4011Y)	Bell System Statistical Manual*				
8.	Total Compensation (A4012Y)	Bell System Statistical Manual*				
9.	Price of Materials (A442Q)	GNP Deflator*				